

Decoherence assisting a measurement-driven quantum evolution process

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We study the problem of driving an unknown initial mixed quantum state onto a known pure state without using unitary transformations. This can be achieved, in an efficient manner, with the help of sequential measurements on at least two unbiased bases. However here we found that, when the system is affected by a decoherence mechanism, only one observable is required in order to achieve the same goal. In this way the decoherence can assist the process. We show that, depending on the sort of decoherence, the process can converge faster or slower than the method implemented by means of two complementary observables.

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During the last two decades a major research effort has been conducted in the emerging field of quantum information theory [1]. Much of this activity started with the observation that the capacity of physical systems to process, store and transmit information depends on their classical or quantum nature [2]. Algorithms based on the laws of Quantum Mechanics show an enhancement of information processing capabilities over their classical counterparts. A large collection of quantum communication protocols such as quantum teleportation [3], entanglement swapping [4], quantum cloning [5, 6] and quantum erasing [7] reveal new forms of transmitting and storing classical and quantum information. Most of these protocols have already been experimentally implemented [8, 9, 10, 11, 12, 13, 14].

A common assumption concerning quantum algorithms and quantum communication protocols is the capacity of performing transformations belonging to a fixed but arbitrary set of unitary transformations together with measurements on a given basis. An interesting application in this context is *quantum information dilution* [15]. Here, an arbitrary unknown state of a two-dimensional quantum system is asymptotically driven onto a particular state by interacting with a finite reservoir of two-dimensional quantum systems. This is implemented by means of a sequence of unitary swapping interactions. Another application is the *measurement-driven quantum evolution* process [16], which is succinctly described, in a two-dimensional Hilbert space, as follows: Suppose that initially the system is in an unknown state ρ_0 . The goal consists in mapping this state onto the known pure target state $|\varsigma\rangle$.

In order to accomplish this task we require a non-degenerate observable $\hat{\varsigma}$ with eigenstates $\{|\varsigma\rangle, |\varsigma_\perp\rangle\}$. So, the target state must belong to the spectral decomposition of $\hat{\varsigma}$.

A measurement of the $\hat{\varsigma}$ observable when the system is in the ρ_0 state projects the system onto the target state

$|\varsigma\rangle$ with probability $p = \langle\varsigma|\rho_0|\varsigma\rangle$. In this case the process succeeds and no further action is required. However, the process fails with probability $1-p$ when the measurement projects the system onto the $|\varsigma_\perp\rangle$ state. Since this state cannot be projected onto $|\varsigma\rangle$ by measuring $\hat{\varsigma}$, it will be necessary to introduce a second observable $\hat{\theta}$ whose non-degenerate eigenstates are denoted by $|0\rangle$ and $|1\rangle$.

Failing the first measurement of $\hat{\varsigma}$, a measurement of $\hat{\theta}$ projects the $|\varsigma_\perp\rangle$ state onto either the state $|0\rangle$ or $|1\rangle$. Since, in principle, both states have a component on the $|\varsigma\rangle$ state, a second measurement of $\hat{\varsigma}$ allows again to project, with a certain probability, onto the target state $|\varsigma\rangle$. So, the success probability p_s of mapping the initial state ρ_0 onto $|\varsigma\rangle$, the target state, after applying the consecutive measurement processes $[M(\hat{\varsigma})M(\hat{\theta})]^N M(\hat{\varsigma})$, that is, a measurement of $\hat{\varsigma}$ followed by N measurement processes each one composed of $\hat{\theta}$ followed by $\hat{\varsigma}$, is

$$p_{s,N} = 1 - \langle\varsigma_\perp|\rho_0|\varsigma_\perp\rangle (1 - 2|\langle 0|\varsigma\rangle|^2 |\langle\varsigma|1\rangle|^2)^N. \quad (1)$$

This expression (1) indicates that the success probability $p_{s,N}$ can be maximized by choosing $|\langle 0|\varsigma\rangle| = |\langle\varsigma|1\rangle| = 1/\sqrt{2}$ that corresponds to the definition of mutually unbiased bases (MUB). It was observed [16] that, for MUB, $p_{s,\max}$ quickly converges to 1 almost independently of $\langle\varsigma_\perp|\rho_0|\varsigma_\perp\rangle$ even if the initial state ρ_0 belongs to a subspace orthogonal to $|\varsigma\rangle$.

In this letter we suppose the constraint that no other observable different from $\hat{\varsigma}$ can be implemented in the process. In this case the decoherence can act as an indirect mechanism which allows taking the state partially out from the $|\varsigma_\perp\rangle$ direction. Therefore, a first measurement of the $\hat{\varsigma}$ observable onto the ρ_0 state projects the state of the system onto the target state $|\varsigma\rangle$ with probability $p_1 = \langle\varsigma|\rho_0|\varsigma\rangle$. However, the process fails with probability $1-p_1$ when the measurement projects the system onto the undesired state $|\varsigma_\perp\rangle$; then a second measurement of $\hat{\varsigma}$ is done again after a time t from the first one. Due to the decoherence mechanism, the $|\varsigma_\perp\rangle$ state evolves to ρ_t and so the probability of projecting it onto the target state is $p_2 = \langle\varsigma|\rho_t|\varsigma\rangle$. The probability p' that this procedure fails in a first measurement of $\hat{\varsigma}$ but is successful

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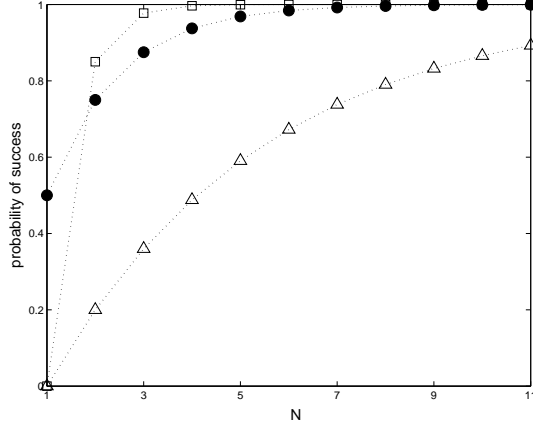


FIG. 1: Probability of success as a function of N with $\langle \varsigma_{\perp} | \rho_0 | \varsigma_{\perp} \rangle = 1$, when two unbiased bases are applied (full circle), and when the decoherence mechanism is considered with $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle = 0.15$ (square) and with $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle = 0.8$ (triangle).

after a second one is $p' = (1 - p_1)p_2$. Let $M(\hat{\varsigma})$ denote a measurement of the $\hat{\varsigma}$ observable; then the success probability in the sequence of measurements $M(\hat{\varsigma})e^{Lt}M(\hat{\varsigma})$ is given by

$$p_{s,2} = p_1 + p' = \langle \varsigma | \rho_0 | \varsigma \rangle + (1 - \langle \varsigma | \rho_0 | \varsigma \rangle) \langle \varsigma | \rho_t | \varsigma \rangle, \quad (2)$$

where e^{Lt} is the generic notation of the unavoidable decoherence mechanism suffered by the two-level system in the state $|\varsigma_{\perp}\rangle$ between two consecutive measurement processes separated by a time t , in such a way that

$$\rho_t = e^{Lt}(|\varsigma_{\perp}\rangle\langle\varsigma_{\perp}|).$$

Similarly, the success probability $p_{s,N}$ of mapping the initial state ρ_0 onto $|\varsigma\rangle$ after applying N consecutive measurement processes of $\hat{\varsigma}$, all of them separated by a time t , is given by

$$p_{s,N} = 1 - \langle \varsigma_{\perp} | \rho_0 | \varsigma_{\perp} \rangle \langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle^{N-1}. \quad (3)$$

In principle the diagonal element satisfies the inequality $0 \leq \langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle \leq 1$ at all time.

Figure 1 shows the probability of success as a function of N when two unbiased bases are applied (full circle), Eq. (1), and when the decoherence mechanism is considered, Eq. (3). In particular we have considered two values of the diagonal element: $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle = 0.15$ (square) and $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle = 0.8$ (triangle).

From Figure 1 we see that, depending on the value of $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle$, the process with decoherence mechanism can converge either faster (square) or slower (triangle) than the procedure by means of unbiased bases (full circle).

We can deduce from Eqs. (1) and (3) that, if

$$\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle \leq 2^{-N/(N-1)}, \quad (4)$$

then the process with decoherence mechanism (DM) converges faster than the procedure realized by means of mutually unbiased bases (MUB), after N measurement procedures. In other words, in order for the DM procedure to converge faster than the MUB process, it is required that at least $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle < 1/2$. The DM process will be fastest when $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle < 1/4$ which corresponds to $N = 2$.

For instance, let us consider a two-level system subjected to pure dephasing decoherence. This effective mechanism [17] is caused by an interaction with at least one boson mode [18] described by the coupling hamiltonian $g(b + b^{\dagger})\sigma_z$, where σ_z is the z -component of the σ spin-1/2 operator with eigenstates $|0\rangle$ and $|1\rangle$. b and b^{\dagger} are the boson annihilation and creation operators respectively, and g gives account of the spin-boson effective coupling strength.

Considering the boson mode to be initially in a thermal state, the $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle$ diagonal element of the ρ_t reduced spin density operator is given by

$$\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle = 1 - 2 \{1 - A_T(t)\} |\langle 0 | \varsigma_{\perp} \rangle|^2 |\langle 1 | \varsigma_{\perp} \rangle|^2, \quad (5)$$

with

$$A_T(t) = \frac{e^{-2(gt)^2}}{1 + \langle n \rangle_T} \sum_{n=0}^{\infty} \left(\frac{\langle n \rangle_T}{1 + \langle n \rangle_T} \right)^n L_n(4g^2t^2),$$

where L_n is the Laguerre polynomial, $\langle n \rangle_T = (e^{\hbar\omega/k_B T} - 1)^{-1}$ is the mean thermal boson number, k_B is the Boltzmann constant, T is the absolute temperature, ω is the frequency of the boson mode, and \hbar is the universal Planck constant. In the low temperature regimen ($T \sim 0$) the (5) element goes to $1 - 2(1 - e^{-2(gt)^2}) |\langle 0 | \varsigma_{\perp} \rangle|^2 |\langle 1 | \varsigma_{\perp} \rangle|^2$ which is always higher than or equal to $1/2$. In the high temperature regime ($\langle n \rangle_T \gg 1$) the (5) element goes to $1 - 2(1 - \delta_{t,0}) |\langle 0 | \varsigma_{\perp} \rangle|^2 |\langle 1 | \varsigma_{\perp} \rangle|^2$ which also is always higher than or equal to $1/2$. It can be easily shown that the density operator of a two-level system under the effect of any pure dephasing mechanism satisfies $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle \leq 1/2$ at all times t and for any $|\varsigma_{\perp}\rangle$ initial state.

As a second example let us consider a two-level system interacting resonantly with a quantized boson mode through the Jaynes-Cummings model [19], $g(b\sigma_+ + b^{\dagger}\sigma_-)$. Similarly to the previous example, the field is initially in a thermal state at T temperature [20]. In this case the $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle$ diagonal element becomes

$$\begin{aligned} \langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle = & \frac{1}{1 + \langle n \rangle_T} \sum_{n=0}^{\infty} \left\{ \left[|\langle 0 | \varsigma_{\perp} \rangle|^2 \cos(\sqrt{n}gt) + |\langle 1 | \varsigma_{\perp} \rangle|^2 \cos(\sqrt{n+1}gt) \right]^2 \right. \\ & \left. + |\langle \varsigma_{\perp} | 0 \rangle|^2 |\langle 1 | \varsigma_{\perp} \rangle|^2 [\sin^2(\sqrt{n}gt) + \sin^2(\sqrt{n+1}gt)] \right\} \left(\frac{\langle n \rangle_T}{1 + \langle n \rangle_T} \right)^n. \end{aligned} \quad (6)$$

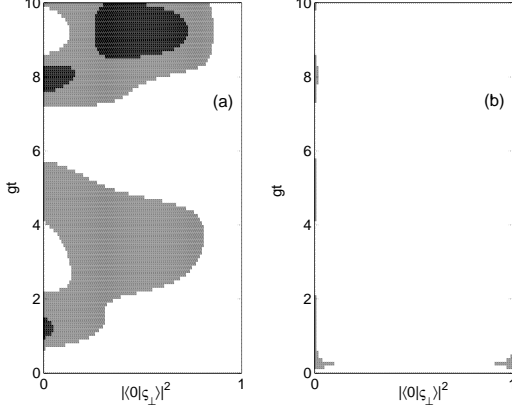


FIG. 2: Diagonal element, $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle$, as a function of gt and $|\langle 0 | \varsigma_{\perp} \rangle|^2$ with: (a) $\langle n \rangle_T = 1$ and (b) $\langle n \rangle_T = 1000$. White means that $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle \geq 1/2$, grey means that $1/2 > \langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle > 1/4$, and black means that $1/4 \geq \langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle \geq 0.06$.

Figure 2 shows the dynamics of the (6) diagonal element for all $|\langle 0 | \varsigma_{\perp} \rangle|^2$ values. We have considered: (a) low temperature with $\langle n \rangle_T = 1$ and (b) high temperature with $\langle n \rangle_T = 100$. Clearly there are some zones of $(|\langle 0 | \varsigma_{\perp} \rangle|^2, gt)$ for which the diagonal (6) element is under the $1/4$ (black zones) which means that at $N = 2$ the decoherence measurement-driven process starts to converge faster than the unbiased bases process. This favorable behavior for the DM procedure is widely and strongly $(1/4 > \langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle > 0)$ present at low temperature whereas for high temperature there are zones where the a weak effect, $(1/2 > \langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle > 1/4)$, is present.

Figure 3 shows the $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle$ diagonal element as a function of gt and $\langle n \rangle_T$, with: (a) $|\langle 0 | \varsigma_{\perp} \rangle|^2 = 0$, and (b) $|\langle 0 | \varsigma_{\perp} \rangle|^2 = 0.5$. For $|\langle 0 | \varsigma_{\perp} \rangle|^2 = 0$, the more efficient behavior zones, this is $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle \leq 1/4$, appear and it remains only for small gt at high $\langle n \rangle_T$ values; meanwhile, for $|\langle 0 | \varsigma_{\perp} \rangle|^2 = 0.5$ a efficient behavior zones appear and it disappear at high $\langle n \rangle_T$ values.

In summary, we have compared two procedures which allow driving an unknown quantum state toward a known pure state by means of von Neumann measurements procedures only. The first process is based on a sequence of measurements of two non-commuting observables. The success probability turns out to be most efficient under the condition that the observables define mutually unbiased bases. The second process which we present here

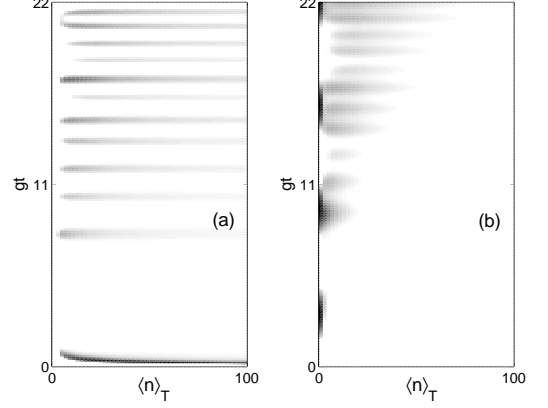


FIG. 3: Diagonal element, $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle$, as a function of gt and $\langle n \rangle_T$ with: (a) $|\langle 0 | \varsigma_{\perp} \rangle|^2 = 1$ and (b) $|\langle 0 | \varsigma_{\perp} \rangle|^2 = 1/2$. White stands for $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle \geq 1/2$ whereas the gray degradation goes from $1/2$ (white) up to ~ 0.18 (black) for graphic (a) and up to $\sim 8 \times 10^{-6}$ (black) for graphic (b).

makes use of a decoherence mechanism instead of a second observable. The decoherence mechanism allows taking the state out from the direction orthogonal to that of the target state. Which of these processes is optimum depends on the $\langle \varsigma_{\perp} | \rho_t | \varsigma_{\perp} \rangle$ value with respect to $2^{-N/(N-1)}$. When this is smaller than or equal to $2^{-N/(N-1)}$, the second procedure converges to 1 faster than the first one after the N measurements. We have shown that the pure dephasing mechanism can not be more efficient than the procedure which makes use of two mutually unbiased bases; whereas a non-dispersive decoherence mechanism described by a Jaynes-Cummings model can be more efficient than the procedure which makes use of two mutually unbiased bases. Thus the decoherence can assist the process and, for a non-dispersive decoherence regimen, it can accelerate the convergence of the process towards the success. It is worth noting that there are some states more stable under the decoherence mechanism than others [21]. Specifically it was found [21] that if one state, for instance the target, has a certain decoherence time scale, its orthogonal direction can have either a higher or a smaller one, depending on the reservoir parameters.

Further studies could involve other natural interactions. For example, one could study a more real model for decoherence mechanism, this is, considering an infinite set of mode near or far from the resonance. It could be also generalized considering an d -dimensional Hilbert

space.

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